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On twinning in smectic crystals

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It is shown that mechanical twinning in smectic crystals is possible. The structure of the boundary of twins for a small disorientation of crystallites is determined. The periodic twin structure, which should appear at the tension of the smectic layer, is proposed.

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The energy of small deformations of a smectic crystal is given by the expression [1]

$$\mathcal{E} = \int \frac{A}{2}\left\{\left(\partial_z u - \frac{(\partial_\alpha u)^2}{2}\right)^2 + \lambda^2(\Delta_\perp u)^2\right\} dV, \quad (1)$$

where u – is the displacement of layers along z axis (in the initial homogeneous undeformed state of the smectic crystal, layers lie in the xy plane, A – is the elastic modulus, λ – is the length parameter, ∂_α – is the gradient vector in the xy plane, and $\Delta_\perp = \partial_\alpha^2$.

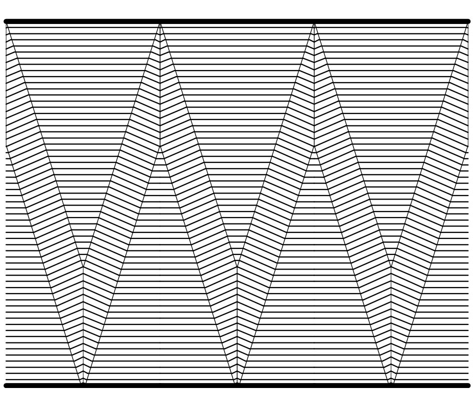
According to Eq. (1), the undeformed state turned by small angle $\theta \ll 1$ in the xz plane (in this case, $\partial_x u = \theta$) corresponds to the derivative $\partial_z u = \theta^2/2$. Let us consider the boundary between the states $\partial_x u = \pm\theta$ ($x \to \pm\infty$) that lies in the yz plane. The quantity $\partial_z u$ is unchanged inside the boundary. The variation equilibrium equation in the problem under consideration reduces to the form

$$\lambda^2 f''' + \frac{\theta^2}{2} f' - \frac{3}{2} f^2 f' = 0, \quad (2)$$

where $f = \partial_x u$. The solution of this equation has the form $f = \theta \cdot \tanh(\theta x/2\lambda)$. The energy of the unit area of this boundary is given by the formula

$$\sigma = 2A\lambda\theta^3/3. \quad (3)$$

The twin structure of smectic crystals must be observed under the conditions of Helfrich instability at strains noticeably larger than the critical value (see the problem in [1, Sect. 45]: the smectic layer of thickness L bounded by solid walls parallel to the smectic layer is extended along the z axis). At very small tensions $\delta L > \delta L_c = 2\pi\lambda$, when δL is about the smectic period, the homogeneous state becomes unstable with respect to the appearance of a periodic structure in the xy plane with wave-number $k_c = \sqrt{\pi/\lambda L}$. At a much larger strain $\delta L \gg \delta L_c$, a twin structure as that schematically shown in the figure should appear.



The parameters of this structure are determined by minimizing the total energy of twin boundaries ($\theta = \varepsilon$, at the vertical boundaries and $\theta = \varepsilon/2$ at the boundaries of the triangular regions). According to geometric consideration, the angle at the vertex of a triangle is equal to ε, the height H of the triangles is related to the structure period d as $\tan(\varepsilon/2) = d/2H$, and the quantity δL is related to the parameter ε as

$$\delta L = (L - H)\left(\frac{1}{\cos\varepsilon} - 1\right).$$

The energy density of the structure proposed above is given by the expression

$$\frac{1}{Ld}\left\{2\frac{L-H}{\cos\varepsilon}\sigma(\varepsilon) + 4\frac{H}{\cos(\varepsilon/2)}\sigma(\varepsilon/2)\right\}. \quad (4)$$

In view of the indicated geometric relationships, at a given tension $\delta L/L$ energy (4) is a function of one parameter ε. When $\delta L \ll L$, angle ε is small. In this case, with the use of result (3), the minimum of energy (4) is found to correspond to $\varepsilon = \sqrt{6\delta L/L}$. In this case, $H = 2L/3$ and

$$d = 2\sqrt{\frac{2L\delta L}{3}}.$$

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1. L.D. Landau and E.M. Lifshitz, Theory of Elasticity, Pergamon, NY (1986)

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